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Proof of a Theorem in Partitions.

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In the *American Journal of Mathematics*, Vol. V, No. 3, p. 257, Professor Sylvester states that the following theorem, although confirmed by a multitude of instances, remains to be proved.

If a square matrix, M_1, of the j^{th} order be formed in which the elements of a diagonal (say of the diagonal which descends from left to right) are all $i+1$, the elements below the diagonal all unity, and those above the diagonal all zeros; and if all possible sets of q rows of this matrix be added together, and the series so formed be added to all possible regularized partitions of content less than n so as to produce a partition, not necessarily regularized, of content n; then the number of times such partition of content n will appear, variously arranged, in P_q, that is in the table formed as above by the addition of the sums of q rows, will be ${}_\mu C_q$ or $\frac{\mu(\mu-1)(\mu-2)\ldots(\mu-q+1)}{1.2.3\ldots q}$ ways, where μ is the number of different elements in the given partition of content n which are greater than i. The proof here offered depends on the consideration of the nature of the series formed from the matrix M_1, and on reversing the order of construction the series obtained from M_1 are of the form

$qqq\ldots(i+q)q-1,\ q-1\ldots(i+q-1)\ldots(i+k+1),\ k,\ k,\ldots(i+k)\ldots(i+1)\,0,\ 0.$

The elements $i+q$, $i+q-1, \ldots i+1$ must be in descending order of magnitude; but there is nothing to prevent q being greater than i; the number of elements in, or the extent of the series is j; the elements, if any, between $i+k+1$ and $i+k$ must be $=k$; but there may be none such: so the elements, if any, to the left of $i+q$ must be q, and the zeros, if any, must be to the right of $i+1$.

Now take a given partition of content n and extent j; if we select q elements greater than i, but containing repetitions, no regularized partition corresponding to these q elements can be obtained, because on subtracting the sequence $i+q$, $i+q-1, \ldots i+1$ from these q elements the remainder would not be regularized; but if the given partition contain μ different elements greater than i, and we select any q of these and ask what must be the mode or modes of arranging the elements of the given partition in order that, after subtracting from it a q-series, derived as above (the elements $i+q$, $i+q-1, \ldots i+1$ being subtracted from the selected q elements) the remainder may be a regularized partition of content less than n, the answer is there is one and only one mode of so arranging the given partition: also there is only one possible subtrahend and one regularized partition of content less than n corresponding to these q elements.

An example will show the mode of arrangement and will lead to the proof. Let 16, 14, 13, 12, 11, 11, 9, 8, 8, 6, 5, 5, 4, 3, 3, 2, 0, 0 be the given partition, n being $= 130$, $j = 18$; let $i = 3$, then $\mu = 10$; let $q = 5$, and the five selected elements be 12, 11, 9, 8, 5; then the elements $i+q$, $i+q-1, \ldots i+1$ are 8, 7, 6, 5, 4; and they must be arranged in descending order; for otherwise the remainders would be unregularized, that is, not in descending order of magnitude.

Again write in descending order the remaining elements of the minuend, viz. 16, 14, 13, 11, | 8, | 6, | 5, | 4, 3, 3, 2, | 0, 0; and it will be found that they must be divided as marked: the 0, 0 must be placed to the right of the 5 in the whole minuend, the 4, 3, 3, 2 between the 8 and the 5; and so on, as indicated below

$$\begin{array}{cccccccccccccccccc}
16, & 14, & 13, & 11, & \overset{*}{12}, & 8, & \overset{*}{11}, & 6, & \overset{*}{9}, & 5, & \overset{*}{8}, & 4, & 3, & 3, & 2, & \overset{*}{5}, & 0, & 0 \\
5, & 5, & 5, & 5, & 8, & 4, & 7, & 3, & 6, & 2, & 5, & 1, & 1, & 1, & 1, & 4, & 0, & 0 \\
\hline
11, & 9, & 8, & 6, & \underset{*}{4}, & 4, & \underset{*}{4}, & 3, & \underset{*}{3}, & 3, & \underset{*}{3}, & 3, & 2, & 2, & 1, & \underset{*}{1}, & 0, & 0
\end{array}$$

The 5 selected elements in the minuend and the corresponding remainders are marked with a star: the 2 in the minuend cannot be placed to the right of the $\overset{*}{5}$; for then on subtracting 0 from 2 the remainder 2 would come to the right of 1, producing an irregularity: 4 in the minuend must come to the right of $\overset{*}{8}$; for $\overset{*}{\text{if}}$ it came to the left 2 or more from 4 would leave 2 or less coming to the left of 3, producing an irregularity: thus it may be shown that 4, 3, 3, 2 must come $\overset{*}{\text{between}}$ $\overset{*}{8}$ and $\overset{*}{5}$, and, if so, then in descending order, because, when equal 1's are subtracted, the remainders are to be regularized; and so on for the remaining sets.

Similarly in other cases after subtracting $i+q$, $i+q-1, \ldots i+1$, respectively, from the q selected elements arranged in descending order, we must arrange the remaining $j-q$ elements in descending order, then divide them into $q+1$ sets (some sets possibly containing no elements) and place the $q+1^{\text{th}}$ set in descending order to the right of $i+1$, the q^{th} set between $i+2$ and $i+1$, and so on; the 1st set from the left being to the left of $i+q$.

If Q_{k+1} and Q_k are the elements from which $i+k+1$ and $i+k$ respectively, are subtracted, then any other element m of the $j-q$ elements will be placed between Q_{k+1} and Q_k if $m \overset{=}{<} Q_{k+1}-(i+1)$ and $\overset{=}{>} Q_k-(i+k)$, that is if m is between $Q_{k+1}-i-1$ and Q_k-i or equal to either of them: any element which is $< Q_1-i$ going to the right of Q_1 and any element which is $> Q_q-i-1$ going to the left of Q_q.

The desired result now follows; for to every selection of q different elements there is one and only one arrangement of the partition of content n appearing in the table P_q: also there is no arrangement of the given partition corresponding to a selection of q elements containing repetitions, that is, in which $i+q$, $i+q-1, \ldots i+1$ are subtracted from those elements. Hence the given partition will appear ${}_{\mu}C_q$ times in the table P_q.